LECTURE NO 15 &16

Gradient of a scalar function

Del operator is given by = **i**

—x + **j** —y+ **k** —z =

grad = **i** —x + **j** —y+

k —z = **i** —— x + **j**

—— y + **k** —— z

——

grad = = **i** +
j + **k**

x y z

Div (Divergence of a vector function)

If **A** = a_1**i** + a_2**j** + a_3**k** , then

div A = .A = **i** —x + **j** —y+ **k** —z .$\left(a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}\right)$

a^1 a_2 a_3

div **A** = **.A** = + +

x y z

Note that

(a) the grad operator acts on a scalar and gives a vector

(b) the div operator acts on a vector and gives a scalar.

Example 4: If A = x^2y**i** xyz**j** + yz^2**k,** then find Div **A.**

Solution:

Div **A** = **.A** = — $(x^2 y)$ — (xyz) — (yz^4)= 2xy xz + 2yz

x y z

Example 5: If A = $2x^2y$**i** $2(xy^2+ y^3$**z**)**j**+$3y^2z^2$**k,** determine **.A** i.e. div **A**. **Solution: A**= $2x^2y$**i** $2(xy^2 + y^3z)$**j** + $3y^2z^2$**k**

.A= a^x + a_y + a_z = 4xy 2(2xy+$3y^2$ **z**) + $6y^2z$ = 4xy 4xy $6y^2z+6y^2z$ = 0

x y z

Such a vector A for which **.A** = 0 at all points, i.e. for all values of x, y, z, is called a solenoid vector. It is rather a special case.

<u>Curl (Curl of a Vector Function)</u>

The curl operator denoted by **A**, acts on a vector and gives another vector as a result. If **A**= a_1**i** + a_2**j** + a_3**k** then curl **A**= **A**.

i.e. curl **A**= **A**= **i** —x + **j** —y+ **k** —z $(a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k})$

i **j** **k**

= x y z

a_1 a_2 a_3

A= **i** — ay_3 — az_2 +**j** — az_1 — ax_3 + **k** — ax_2 — ay_1

Curl **A** is thus a vector function.

Stokes' Theorem

In words we can state Stokes' theorem as the line integral of the tangential component of a vector function $\vec{A}$ taken around a simple closed curve C is equal to the surface integral of the normal component of the curl of $\vec{A}$ taken over any surface S having C as its boundary.

Statement

It states that if S is an open, two sided surface bounded by a simple closed curve C, then if $\vec{A}$ has 20
continuous first partial derivatives

$$\oint_C \vec{A}.d\vec{r} = \iint_S (\nabla \times \vec{A}).\hat{n}dS$$

Where C is traversed in the positive direction.

Proof

If $\vec{A}$ is expressed as $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$, then the divergence theorem can be written as

$$\iint_S \nabla \times \left(A_1\hat{i} + A_2\hat{j} + A_3\hat{k}\right)dS = \oint_C A_1dx + A_2dy + A_3dz$$

We will prove this theorem for a surface S which has the property that its projection on the xy, yz and zx planes are regions bounded by simple closed curves as shown in figure.